

MTH 301: Group Theory

Practice Assignment II

1. Classify all homomorphisms from:

(a) $\mathbb{Z}_m \rightarrow \mathbb{Z}_n$

(b) $\mathbb{Z}_m \rightarrow \mathbb{Z}$

(c) $\mathbb{Z} \rightarrow \mathbb{Z}_n$

2. Establish all the assertions in Examples (a) and (b) in 5.2 (xi) of the lesson plan.

3. Show that the following statements pertaining to the semi-direct product $G \rtimes_{\psi} H$ of two groups are equivalent.

(a) ψ is the trivial homomorphism.

(b) $G \rtimes_{\psi} H = G \times H$.

(c) $G \trianglelefteq G \rtimes_{\psi} H$.

4. Let G be a group, $N \trianglelefteq G$, and $H \leq G$ with $H \cap N = \{1\}$. Then show that

$$NH \cong H \rtimes_{\psi_c} N,$$

where ψ_c is permutation representation afforded by the action $H \curvearrowright N$ by conjugation i.e. the action given by $(h, n) \mapsto hnh^{-1}$.

5. Find the action $\mathbb{Z}_2 \curvearrowright \mathbb{Z}_n$ that affords a permutation representation ψ so that

$$\mathbb{Z}_2 \rtimes_{\psi} \mathbb{Z}_n \cong D_{2n}.$$

[Hint: See Example 5.2 (xi) b.]